LSU Dept. of Physics and Astronomy
Qualifying Exam
Thermo and Statistical Mechanics Question Bank
(11/2025)

1. This question has two independent parts:
   (a) Show that the enthalpy H can be related to the Gibbs free energy as

$$H = -T^2 \left[\frac{\partial}{\partial T} \left(\frac{G}{T} \right)_P \right]$$

   (b) Show that

$$C_V = -T \left(\frac{\partial^2 F}{\partial T^2} \right)_V$$

2. Mercury has a radius of 2.44×10^6 m and is at a distance of 5.8×10^{10} m from the Sun, which has a radius of 7×10^8 m. The power output from the Sun is 4×10^{26} W. Note that one side of Mercury always faces the Sun. Treating Mercury as a black body:
   (a) Find the total power absorbed by Mercury from the Sun. (2 pts)

   (b) If Mercury is in thermodynamic equilibrium, it will emit the same total power as it receives from the Sun. Assuming that the temperature of the "hot side" of Mercury is uniform, find this temperature. (4 pts)

   (c) What is the ratio of peak frequency of the radiation emitted by the Sun to that of Mercury? (4 pts)

3. A box of volume V has three partitions filled with ideal gases with initial temperature T = 300 K and pressure P = 2 atm. In the first partition there is a kilomole of helium gas, in the second there are two kilomoles of neon gas, and three kilomoles of argon gas are in the third partition. After the partitions are removed and the gases diffuse after some time in the entire box,
   (a) Find the final individual pressures of each component and the total pressure. (3 pts)

   (b) Find the change in Gibbs free energy G in the mixing process. (3 pts)

(c) Find the change in entropy of the system in the mixing process. (4 pts)

4. This question has two independent parts:
   (a) Show that the internal energy U can be related to the Helmholtz function F as

$$U = -T^2 \left[\frac{\partial}{\partial T} \left(\frac{F}{T} \right)_V \right]$$

   (b) Show that

$$C_P = -T \left(\frac{\partial^2 G}{\partial T^2} \right)_P$$

5. This question has two independent parts:
   (a) Find the melting point of Aluminum at a pressure of 10^7 Pa if at atmospheric pressure (10^5 Pa) it melts at 550 K. Note that while it melts its density decreases from 3×10^3 kg/m^3 to 2.9×10^3 kg/m^3. Latent heat of fusion of aluminium is 24×10^3 J/kg.

   (b) The internal energy of a system at a fixed volume is found to depend on the temperature T as $E(T) = aT^2 + bT^4$, where a and b are constants. Find the expression of entropy.

6. In Fig. 1, a liquid is in contact with a reservoir at room temperature T. When the object of mass m falls through a height h, the paddle inside the liquid moves and this causes an increase in the temperature of the liquid. The adiathermal contact between the system and the reservoir causes the heat flow from the former to the latter, as a result, the state of the system is not affected. Giving detailed explanations, find
   (a) the change in entropy of the water
   (b) the change in entropy of the paddle
   (c) the change in entropy of the mass m
   (d) the change in entropy of the reservoir.

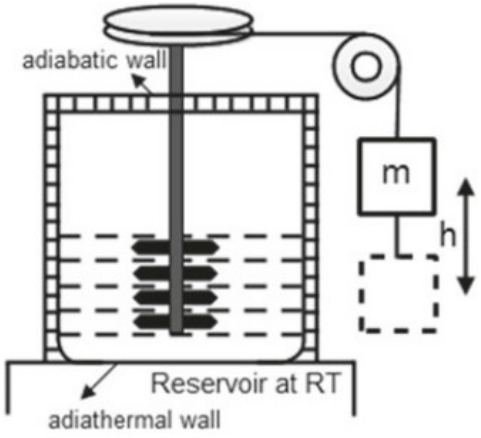

Figure 1: Figure shows a Joule paddle. RT refers to room temperature.

7. (a) If n moles of an ideal gas expands isothermally and reversibly to twice its original volume. Calculate the change in entropy of the gas, and of the universe. What happens if the gas undergoes free expansion?

   (b) The specific volume of a liquid is much less than that of its vapors. If we assume that the vapor obeys the equation for an ideal gas show that pressure depends exponentially on temperature.

8. (a) A cylinder which is thermally insulated from the environment contains a mole of ideal gas with constant heat capacity C_V. The cylinder has a piston which can vertically without friction. Pressure P_1 is applied to the piston, and then the pressure is changed abruptly to P_2. As a result, the gas volume changes adiabatically. Find the temperature T_2 and the volume V_2 after the thermodynamic equilibrium has been reached.

   (b) After the thermodynamic equilibrium has been established in above part, if the pressure is abruptly reset to its original value P_1 find the final values of the temperature T_f and the volume V_f once the thermodynamic equilibrium is established again. Use the first law of thermodynamics and the adiabatic equation to compute the difference in temperatures ($T_f - T_1$). Discuss the sign and the relative magnitude of the temperature difference with respect to the change in pressure.

9. 1000 joules of heat are added at constant pressure of 1 atm to an ideal monoatomic gas. The system expands in the process and its temperature rises 10 K.

   (a) Find the number of moles present (2 pts).

   (b) Find the increase in volume (3 pts).

   (c) Find the entropy change in terms of the initial temperature (5 pts).

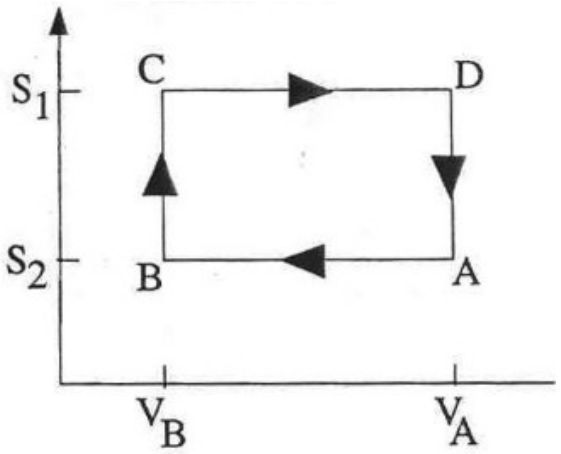

Figure 2: Figure shows the Otto cycle. Gas is compressed adiabatically in A → B, gas is heated isochorically in B → C, gas is expanded adiabatically in C → D, and gas is cooled isochorically in D → A.

10. Figure 2 depicts the approximate behavior of a gasoline engine which is similar to that of the Otto cycle. Compute the efficiency for an ideal gas (with temperature-independent heat capacities) as a function of the compression ratio V_A/V_B, and the heat capacity per particle C_V.

11. (a) Show that, if the temperature is constant, the pressure of an ideal gas in a uniform gravitational field decreases with height according to the formula

$$p(z) = p(0)e^{-mgz/k_B T}.$$

    (b) The result from the preceding part assumed that temperature is constant, so that equilibrium is attained by the transfer of heat. Now assume heat is not exchanged to achieve equilibrium, but via an *adiabatic* process that holds the quantity pV^γ constant (γ being the adiabatic index). What is $p(z)$ for this case?

12. The thermodynamic identity is:

$$dE = T\,dS - pdV + \mu dN. \qquad (1)$$

We obtain the definition of temperature, pressure and chemical potential by holding two of (E, S, V, N) fixed. For example, by holding N and V fixed ($dN = 0, dV = 0$) we get $1/T = \partial S/\partial E|_{N,V}$.

(a) Generalizing this idea, how many formulas of this form can be derived by holding two quantities fixed in Eq. (1)? Derive them all.

(b) Three of the formulas from part (a) are just the definitions of T, μ and p, but the rest of the formulas are nontrivial. Demonstrate that the latter are valid using the triple product rule

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x = -1.$$

13. The internal energy of a thermodynamic system, $E(S, V, N)$ satisfies the thermodynamic identity

$$dE = T\,dS - pdV + \mu dN.$$

(a) Derive the thermodynamic identities for the Helmholtz free energy $A = E - TS$, the Gibbs free energy $G = E + pV - TS$, and the thermodynamic potential $\Omega = -A + \mu N$.

(b) Using the fact that Ω is extensive, show that $\Omega = pV$.

14. A steel rod with length $l = 20$ cm and cross-section $A = 3$ cm^2 is heated at one end to $T = 300$°C, and touches a block of ice at $T_i = 0$°C with the other end. Assuming that all the heat is transferred along the rod, with no losses through the sides, estimate the mass of ice that melts in $t = 5$ min. Latent heat of melting for ice is 80 cal/g, the heat conductivity of steel is $k_t = 0.16$ cal/(s·cm·°C).

15. A monoatomic ideal gas is expanded under such conditions that $PV^2 = \text{const}$. What is the heat capacity per mole of the gas under this process?

16. You know the relationship between the entropy, S, internal energy, U, number

of particles, N, and volume, V, of a substance,

$$S = A\,[UNV]^{1/3},$$

where A is a constant.

(a) Verify that entropy is an extensive quantity according to this definition, and obtain the units for the constant A.

(b) Derive the internal energy as a function of temperature, volume, and the number of particles, $U(N, T, V)$.

(c) Derive the equation of state, i.e. the relationship between pressure, P, volume, temperature, and the number of particles; Verify that the pressure is an intensive quantity.

(d) Derive the heat capacity at constant volume, C_V.

(e) Derive the chemical potential $\mu(T, P)$.

17. Consider the following adsorption problem (Fig. 3). A surface has $N \gg 1$ "best" binding sites, and $M \gg 1$ "good" binding sites for a particular molecule. Each site can be occupied by at most one molecule. The energy of a molecule at a "good" site is ε higher than then energy at a "best" site. You initially exposed the surface to a gas of these molecules for a long time at low temperature, so that all N best sites are occupied by the molecules, and all the good sites are empty, as shown at left. Now you stopped the exposure, and increased the temperature, so that n molecules may be promoted to "good" sites, while $N–n$ molecules stay at the "best" sites, as shown at right. The energy of this configuration is obviously $U = n\varepsilon$.

(a) For a fixed number of adsorbed molecules the energy is simply determined by the number of molecules at the "best" sites. Write the expression for the degeneracy (multiplicity) of this state, i.e. the state with $N–n$ molecules at "best" sites, and n molecules at "good' sites.

(b) Using the definition of temperature via the entropy, derive the general equation for n as a function of temperature. You don't have to solve it yet, but it should only contain n and known quantities, i.e. be simple and solvable.

(c) Find $n(T)$ when there is an equal number of different sites, $N = M$. Simplify the final expression as much as you can.

(d) Show that if $N \gg M \gg 1$ there are two temperature regimes: low temperature, when the occupation of M-type sites is low, and high temperature, when this occupation is high. Derive the *approximate and simple* expressions for $n(T)$ in both limits and roughly estimate the *crossover temperature* between the two regimes. The easiest way to proceed is to look at the general equation you derived, and drop some terms based on physical and mathematical reasoning.

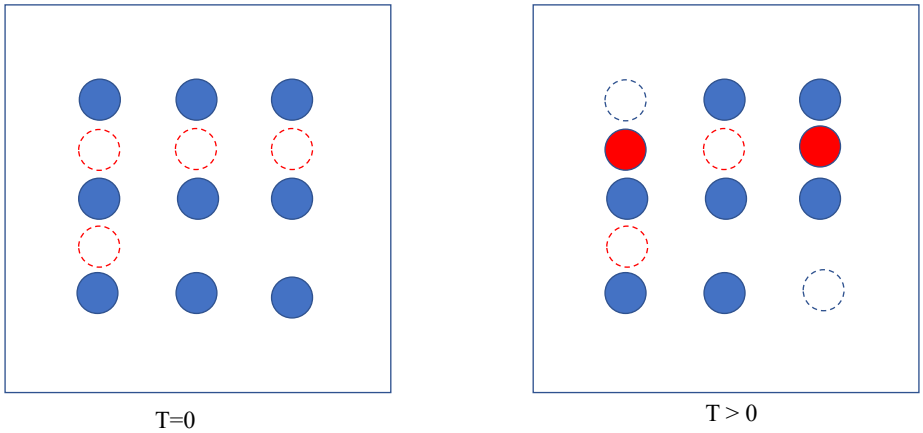

Figure 3. Adsorbed atoms on the surface. Left panel: low temperature, all atoms (blue circles) are at "best" sites, while "good" sites are empty (dashed circles). Right panel: higher temperature: atoms shown in red moved from "best" to "good" sites.

18. Consider a system that has four energy eigenstates for single particles: two distinct ones with energy E_0, and two other distinct levels with energy $2E_0$. This system contains 3 non-interacting bosons. What is the temperature range (if any) where we are more likely to find the system in the first excited (net) energy state rather than in the ground state?

19. Consider a system consisting of N one-dimensional simple-harmonic quantum oscillators at temperature T. Each oscillator has energy $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$, where $n = 0, 1, 2, \cdots$.
(a) What is the mean energy of the system?

(b) Under what conditions does the system obey the equipartition theorem? Explain why.

20. Consider a classical N-particle system with Hamiltonian

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i=1}^{N} \frac{k_i}{2} q_i^2,$$

at temperature T. For this problem, you may use the formulas

$$\int_{-\infty}^{\infty} dx\, \mathrm{e}^{-ax^2} = \frac{\sqrt{\pi}}{\sqrt{a}},$$
$$\int_{-\infty}^{\infty} dx\, x^2 \mathrm{e}^{-ax^2} = \frac{\sqrt{\pi}}{2a^{3/2}}.$$

(a) Show that:

$$\frac{\langle p_i^2 \rangle}{2m_i} = \frac{1}{2} k_{\mathrm{B}} T.$$

(b) Derive a similar relation, involving q_i.

21. What does a surface of constant energy look like in the phase space of an oscillator of frequency ν? Find the volume $\Gamma(E)$ of phase space below E for a classical oscillator of such a frequency. Then find the number of quantum states $\Omega(E)$ with energy below E for the same oscillator and show that when E is large, $\Gamma/\hbar \approx \Omega$.

22. (a) For an oscillator with mass m and angular frequency ω, calculate the partition function (i) classically and (ii) quantum mechanically.

(b) With the quantum partition function above, find the internal energy, entropy, and heat capacity of a system of N such oscillators as a function of temperature T.

23. An ideal gas consisting of N particles of mass m obeying classical statistics is enclosed in a tall cylinder placed in a constant gravitational field, and is in thermal equilibrium. Calculate the classical partition function, Helmholtz free

energy, mean energy, and heat capacity of the system. Comment on the heat capacity relative to an ideal gas at constant volume.

24. The energy difference between the lowest atomic state $^1S_0$ and first excited $^3S_1$ state of the helium atom is 159,843 cm^{-1}. The superscripts of the spectroscopic states denote the degeneracy. Evaluate the fraction of excited states in He gas at 6000 K.

25. (a) Find the ratio of the number of ortho-hydrogen and para-hydrogen molecules in a H_2 gas. What would this be for heavy hydrogen D_2? The nuclear spin of H is 1/2 and of D is 1, and ortho has the greater spin weight than para.

(b) The rate of ortho-para conversion is so small that the two forms can be separated as though they are different kinds of gases. Calculate the specific heat at low temperatures and show that para-hydrogen has the larger specific heat.

26. A crude estimate of the surface temperature of the earth is to assume that the clouds reflect a fraction of all sunlight, the rest being absorbed by the earth and reradiated. Treating the sun as blackbody at a temp T = 5800K, find the surface temp of the earth. You may assume the earth is an ideal absorber and that the rotation of the earth allows it to emit in all directions. The radius of the sun is 6.96 x 10 km and that of the earth is 6,400 km. The mean distance between the sun and the earth is 1.5 x 10 m.

27. A simplified model of diffusion consists of a one-dimensional lattice, with lattice spacing a, in which an "impurity" makes a random walk from one lattice site to an adjacent one, making jumps at time intervals Δt. After N jumps, the atom has taken N_1 steps to the right and N_2 steps to the left with $N = N_1 + N_2$. It is now located at $x = a\,(N_1 - N_2)$.

a) Find the probability the atom is at x, after N steps, given $N_1, N_2 \gg 1$.

b) If a, Δt are taken infinitesimal the probability in (a) satisfies the diffusion equation

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}$$

Find an expression for D in the terms of a, Δt.

28. Show that : $C_p = C_V + TV\alpha^2 / k_T$, where C_p and C_v are the heat capacities at constant pressure and volume, respectively, $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ is the coefficient of thermal expansion, and $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$ is the isothermal compressibility.

29. Lead has a molar mass of 207.2 g/mole. At 25°C and 1 atmosphere of pressure, it has an isothermal bulk modulus of B = 1.6 x 10^{10} Pa, a mass density ρ = 11.4 g/cm^3, and a coefficient of thermal expansion of 87 x 10^{-6}/°C. Its specific heat at constant pressure C_p = 128 J/kg-°C.

a) How big is the difference between C_p and its constant volume specific heat C_V?

b) The Law of Dulong and Petit states that the heat capacity of any solid at room temperature arises from the vibrations of the atoms (3N degrees of freedom), which can be calculated by treating the vibrations as a set of 3N classical harmonic oscillators. Does the Law of Dulong and Petit describe C_p or C_V? What would you predict the heat capacity of lead to be if this law is correct?

c) Find the Debye temperature of lead. How does the specific heat of lead vary with temperature for temperatures well below the Debye temperature?

30. Consider a monoatomic ideal gas of mass density ρ at temperature T, whose atoms have mass m. The number of atoms with velocities $\vec{v}$ in the velocity space volume element $d^3\vec{v}$ is given by the Maxwell-Boltzmann distribution

$$n(v)d^3v = \frac{\rho}{m}\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left[\frac{-m(v_x^2 + v_y^2 + v_z^2)}{2k_B T}\right] d^3v$$

a) What is the average velocity v_{avg}?

b) Derive the distribution of speed $P(v)dv$.

c) What is the most probable speed v^*?

d) Obtain expressions for the average $\bar{v}$, and root-mean-square speed v_{rms} and v^*, and rank them in increasing order.
[Hint: $\int_0^\infty x^n e^{-x^2} dx = \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)$.]

31. A certain material is completely specified by its volume V and temperature T. It has an equation of state $p = AT^4$, where A is a constant independent of the volume. The heat capacity at fixed volume is measured to be BVT^3.

a) From dimensional analysis, B and A have the same units. Show that $B = 12A$.

a) Find the entropy of this material as a function of V and T.

c) If this material is cooled adiabatically and reversibly from 20K to 10K, by how much does the volume change?

32. Consider an ideal Fermi gas of spin 1/2 particles in 3-dimensional box. The number of particles per unit volume is n, the mass of the particles is m, and the energy of the particles is the usual $E = p^2/2m$. Assume that the temperature is quite low.

a) Find formulas for the Fermi energy E_F, Fermi wavevector k_F, and Fermi temperature T_F in terms of m, n and constants such as h and k_B.

b) Find the total energy of the gas at zero temperature.

c) Show that the heat capacity at low temperature is proportional to T.

33. A collection of N spin 1/2 atoms are fixed in a solid. The atoms do not interact with each other. The magnetic moment of each atom is $\pm\mu_0$. If a magnetic field H is applied to the solid, each atom has an energy of $\pm\mu_0 H$.

a) Find the mean energy $\langle E \rangle$ in a magnetic field at a given temperature T.

b) Find the entropy of this collection.

c) The magnetization m of a solid is defined as the net magnetic moment per unit volume. The average magnetic moment is defined via $\langle E \rangle = - \langle M \rangle H$. For noninteracting moments, the magnetization typically obeys a Curie Law where $m = \chi_0 H/T$ for vanishingly small H. Find the value of the constant χ_0 for this problem.

34. A collection of N bosons is contained in a volume V. The spin of the particles is 0.

a) Find the temperature at which Bose condensation occurs.

b) Find how the number of particles in the lowest energy state varies with temperature below the condensation temperature.

35. The average energy of a system in thermal equilibrium is $\langle E \rangle$. Prove that the mean square deviation of the energy, $\left\langle \left(E - \langle E \rangle\right)^2 \right\rangle$ is given by

$$\left\langle \left(E - \langle E \rangle \right)^2 \right\rangle = k_{\mathrm{B}} T^2 C_V$$

where C_V is the heat capacity of the system at constant volume and k_B is the Boltzmann constant. Use this result to show that the energy of a macroscopic system may ordinarily be considered constant when the system is in thermal equilibrium.

36. Consider a container of volume 100 cm^3 containing a classical ideal gas at 1 atm pressure and 350K.

a) Find the number of particles.

b) Compute the mean kinetic energy of a particle in the gas.

c) Suppose one counted the number of particles in a small subvolume of size 0.1 micron on a side. What is the probability of finding no particles in this volume?

37. The pressure in a vacuum system is 10^{-3} mm Hg. The external pressure is 1 atm at 300 K. This is a pinhole in the vacuum system of area 10^{-10} cm^2. Assume that any molecule entering the pinhole goes through. Use an average molecular weight of air as 29 amu.

a) How many molecules enter the vacuum system each hour?

b) If the volume of the system is 2 liters, by how much does the pressure rise in 1 hour?

c) How long does it take for the pressure to rise to 750 mm Hg? Note: this is close to the pressure outside the vacuum tank.

38. The diatomic molecule HD has a set of rotational energy levels $E_\ell = k_B\theta_r \ell(\ell+1)$ which are $(2\ell+1)$ degenerate, and a vibrational spectrum $E_v = (k_B\theta_v)(n_v + \frac{1}{2})$. θ_r is called the rotational temperature, and θ_v the vibrational temperature.

a) Assuming the molecules do not interact among themselves, evaluate the partition sum for *N* atoms confined to a volume *V*. You may not be able to compute all the sums in closed form.

b) Evaluate the Helmholtz Free energy *F(T, V, N)* in the limits $T << \theta_r \theta_r << T << \theta_v$, and $T >> \theta_v$. It will help to use the fact that when $k_B T$ is much smaller than the spacing of the energy levels, the sum can be computed as an integral.

c) Sketch, as accurately as you can, the behavior of $C_v(T)$ from 25K to 5000K, using a logarithmic scale in temperature. Assume that $\theta_r = 100K$ and $\theta_v = 3000K$.

39. Consider an equilibrium gas consisting of two types of atoms of masses m_1 and m_2, both obeying Maxwellian velocity distribution corresponding to the same temperature T,

$$f_i(\vec{v}_i)d\vec{v}_i = \left(\frac{m_i}{2\pi kT}\right)^{3/2} \exp[-m_i v_i^2 / 2kT]d\vec{v}_i,$$

where m_i is the mass of particle type i, and the f_i are the corresponding velocity distributions. Using the well-known identity

$$m_1\vec{v}_1^2 + m_2\vec{v}_2^2 = (m_1 + m_2)\vec{V}^2 + \mu\vec{v}^2,$$

where $\vec{V}$ is the center-of-mass velocity, $\vec{v}$ is the relative velocity and $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass, show that the distribution of relative velocities between the two types of atom is also Maxwellian, with distribution

$$f_{1,2}(\vec{v})d\vec{v} = \left(\frac{\mu}{2\pi kT}\right)^{3/2} \exp[-\mu v^2 / 2kT]d\vec{v}\,.$$

40. An object of heat capacity *C* is used as the cold thermal reservoir by a Carnot engine. The hot reservoir has an infinite heat capacity. During the operation of the engine to produce work, the temperature of the cold reservoir will slowly rise. Assume the starting temperature of the cold reservoir is T_c and the constant temperature of the hot reservoir is T_h.

a) To what temperature will the cold reservoir rise before the engine ceases to produce work?

b) How much heat flows into the cold reservoir until the engine stops producing work?

c) What is the total amount of work the engine can produce before the process stops?

41. A certain gas has an equation of state

$$P = b + \frac{nRT}{V}.$$

a) Find the value of $C_p - C_V$, where C_p and C_V are the heat capacities at fixed pressure and volume.

b) Show that the constant volume heat capacity is not dependent on the volume. Specify how the heat capacity can depend on *n* and *T*.

c) This gas undergoes a process at fixed temperature where the volume is changed from V_i to V_f. Find the change in entropy in terms of the variables n, R, T, V_f and V_i.

42. Consider a classical gas composed of N particles of mass m that possess a permanent electric dipole moment p. The gas is subject to a uniform electric field of size E in the x-direction and is enclosed in a container of size V. The potential energy of a dipole is $-\vec{E}\cdot\vec{p}$.

a) Find the partition function.

b) Find the net polarization of the sample as a function of field strength, pressure, and temperature. The polarization P is defined as the net dipole moment per unit volume.

43. The energy spectrum of neutrinos (spin ½ massless particles) is $E=pc$. For a collection on N neutrinos confined to volume V, calculate

a) The Fermi wave vector.

b) The Fermi energy.

c) The total energy at T = 0.

d) The compressibility of the neutrino gas.

44. Consider an ideal Fermi gas of spin -1/2 particles in a 3-dimensional box. The number of particles per unit volume is n, the mass of the particles is m, and the energy of the particles is the usual $E=p^2/2m$. Assume that the temperature is quite low.

a) Find formulas for the Fermi energy E_F, Fermi wavevector k_F, and Fermi temperature T_F in terms of m, n and constants such as h and k_B.

b) Find the total energy of the gas at zero temperature.

c) If the magnetic moment of the particles is u_e, show that the paragmagnetic susceptibility x for low fields in the limit of zero temperature is given by

$$x = \frac{3n\mu_e^2}{2E_F}$$

45. Consider a system consisting of two particles, each of which can be in any one of three quantum states of respective energies 0, ε, and 3ε. The system is in contact with a heat reservoir at temperature T.

a) Write an expression for the partition function Z if the particles obey classical Maxwell-Boltzmann statistics and are considered distinguishable.

b) Write a similar expression if the particles obey Bose-Einstein statistics.

c) Write a similar expression if the particles obey Fermi-Dirac statistics.

46. A box of volume V hold blackbody radiation at a temperature T.

a) Show that the amount of energy stored in a small range of frequencies $(f, f+df)$ is given by

$$E_f\, df = \frac{3af^3}{e^{bf}-1} df \ ,$$

and find all expressions for a and b in terms of V, T, K_B, h, c.

b) The Wien displacement law describes the relationship between the peak intensity in the blackbody energy distribution (1) and the temperature. Derive this relationship. Note: the function $x^3/(e^x - 1)$ has a maximum at $x = 2.82$ approximately.

c) Find the total energy of the blackbody radiation and how it depends on V, T, h, c, k_B.

47. Consider a degenerate electron gas in which all electrons are considered to be highly relativistic, so that their energy $\epsilon = cp$ with p the magnitude of the momentum vector.

a) Calculate the density of states for this system.

b) Derive a relation, at zero temperature, between the particle density N/V and the Fermi energy ϵ_F for this system.

c) The internal energy of a nonrelativistic zero temperature 3D electron gas satisfies $U = \frac{3}{5}N\epsilon_F$. Derive the corresponding relation for the present relativistic electron gas.

48. According to quantum mechanics the possible energy levels of a simple harmonic oscillator are given by

$$E_n = (n + \frac{1}{2})\hbar\omega ,$$

where n= 0, 1, 2, 3, $\cdots$, $\hbar$ is the Plank's constant (divided by 2π), and ω is the angular frequency of the oscillation.

a) Calculate the average energy of the oscillator at a fixed temperature *T*.

b) The Einstein solid consists of 3*N* such harmonic oscillators, all at the same frequency ω. Find the heat capacity at constant volume of the Einstein solid and describe its behavior at high temperature and low temperature limits, respectively.

49. Derive the density of states for a uniform gas of bosons confined to an area A in two spatial dimensions. Show that Bose-Einstein condensation is not possible for this system. Finally, determine the system chemical potential, as a function of temperature and sketch it.

50. A liter of air, initially at room temperature and atmospheric pressure, is heated at constant pressure until it doubles in volume. Calculate the increase in entropy during this process.

51. Using the thermodynamic identities, prove one of the Maxwell's relations:

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

52. Suppose you heat 200 g of water from 20°C to 100°C by contact with a heat reservoir at 100°C. (a) Find the heat absorbed by the water. (b) Find the change in entropy of the water. (c) Find the change in entropy of the universe.